COMPARATIVE ANALYSIS OF PRECISION OF SINE SIGNALS INTERPOLATION USING OF THE FIFTH AND SEVENTH ORDER POLYNOMIAL INTERPOLATION KERNELS

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Abstract

In the first part of this paper, one-parameter polynomial interpolation kernels of the fifth and seventh order are described. After that, an Experiment, which was realized with the aim of determining the higher precision of the interpolation of sine signal, between the fifth and seventh order kernels, was described. The interpolation accuracy was analyzed using MSE. The optimal value of the kernel parameter is determined by minimizing the MSE. The results of the Experiment are presented using graphs and tables. At the end, a comparative analysis of the results was performed. Based on the analysis, a recommendation on which kernel should be used, was made.

Keywords: convolution, interpolation, interpolation kernel, kernel parameter.

1. INTRODUCTION

In order to increase the speed of the interpolation, as well as the precision of interpolation in digital signal processing and digital image processing, the convolutional interpolation is intensively used. Convolutional interpolation is performed using the interpolation kernel [1]. The ideal interpolation kernel is with form $r(x)=\sin \pi x/\pi x$ (*sinc* function), where is x = -∞ to +∞. The spectral characteristic of the ideal interpolation kernel is a box function. In the practical realization of the ideal kernel, the basic problem is infinite length of the kernel. That is why there is a need to shorten the length of the interpolation kernel. Shortening the length of the kernel causes a deviation of the spectral characteristic in relation to the box function. The deviation manifests itself in the form of a ripple of the spectral characteristic in the pass-band and stop-band. As a consequence of the described characteristics of the physically realizable kernel, an increase in the interpolation error appears. That is why intensive work is being done on the construction of the interpolation kernel with finite length. The *sinc* function is approximated by polynomial functions of low order (n < 9). The implementation of the polynomial kernels enables lower numerical complexity of the interpolation algorithms, and thus, higher interpolation speed. In order to minimize the interpolation error, and adapt it to the specific signal being interpolated, the kernel is parameterized [2]. A one-parameter (1P), third-order polynomial interpolation kernel was described by Keys in paper [3]. After that, in the scientific literature, this kernel was called the 1P Keys kernel. By choosing the value of the kernel parameter, α, it is possible to minimize of the interpolation error. Therefore, optimization of the kernel parameter, α_{opt}, was performed. Further scientific activities in the field of polynomial kernels went towards reducing of the interpolation error, by creating two-parameter (2P) Keys kernels [4]. For now, the highest precision of the interpolation of the audio signals (musical signals, speech signals,...) has been achieved using the three-parameter (3P) Keys kernel [5]. The polynomial 1P interpolation kernels of the fifth and seventh order are shown in [6].

In this paper, experimental results of interpolation precision, in the case of implementation of the 1P interpolation polynomial kernels of the fifth and seventh order, are shown. An Experiment was carried out in which the Test signal Base was formed. The Test signals, of time sinusoidal form, based on mathematical definitions are created. Sine Test signals are created by superposition of the Sine signal with the basic, fundamental, frequency f_0 and the sine signal with a frequency nf_0 (n = 2, ..., 10). In this way, the Sine test signal has become harmonic, and thus it becomes a synthetic music signal. The Test signal Base is created from Sine test signals that correspond to musical signals G1 - G7. The precision of interpolation, using mean square interpolation error, *MSE*, was analyzed. The results are presented using tables and graphs. Finally, by applying a comparative analysis, the precision of interpolation of the 1P kernels of the fifth and seventh order was evaluated.

The further organization of this paper is as follows. In Section 2, 1P kernels of fifth and seventh order are described. Experimental results and comparative analysis of the results are presented in Section 3. Section 4 is the Conclusion.

2. 1P INTERPOLATION KERNEL

2.1 Fifth order 1P interpolation kernel

The convolutional, 1P interpolation kernel of the fifth order, length L = 6, is defined in the paper [1]:

$$r_5(x)=\begin{cases}\left(10\alpha-\frac{21}{16}\right)|x|^5-\left(18\alpha-\frac{45}{16}\right)|x|^4 & \\ +\left(8\alpha-\frac{5}{2}\right)|x|^2+1, & |x|\le 1\\ \left(11\alpha-\frac{5}{16}\right)|x|^5-\left(88\alpha-\frac{45}{16}\right)|x|^4 & \\ +(270\alpha-10)|x|^3-\left(392\alpha-\frac{35}{2}\right)|x|^2 & 1<|x|\le 2\\ +(265\alpha-15)|x|-(66\alpha-5), & \\ \alpha|x|^5-14\alpha|x|^4+78\alpha|x|^3 & \\ -216\alpha|x|^2+297\alpha|x|-162\alpha, & 2<|x|\le 3\\ 0 & |x|>3\end{cases} \quad (1)$$

where α is the kernel parameter.

2.2 Seventh order 1P interpolation kernel

The convolutional, 1P interpolation kernel of the seventh order, length L = 8, is defined in the paper [1]:

$$r_7(x)=\begin{cases}a_7|x|^7+a_6|x|^6+a_5|x|^5+a_4|x|^4 & |x|\le 1\\ +a_3|x|^3+a_2|x|^2+a_1|x|+a_0, & \\ b_7|x|^7+b_6|x|^6+b_5|x|^5+b_4|x|^4 & 1<|x|\le 2\\ +b_3|x|^3+b_2|x|^2+b_1|x|+b_0, & \\ c_7|x|^7+c_6|x|^6+c_5|x|^5+c_4|x|^4 & 2<|x|\le 3\\ +c_3|x|^3+c_2|x|^2+c_1|x|+c_0, & \\ d_7|x|^7+d_6|x|^6+d_5|x|^5+d_4|x|^4 & 3<|x|\le 4\\ +d_3|x|^3+d_2|x|^2+d_1|x|+d_0, & \\ 0, & |x|>4\end{cases} \quad (2)$$

The coefficients of the seventh order kernel are determined in accordance with the general conditions that apply to the interpolation kernel, which are defined in [1]:

$$\begin{cases}a_0=1, & a_1=0,\\ a_2=-384\alpha-\frac{1393}{578}, & a_3=0,\\ a_4=760\alpha+\frac{1960}{867}, & a_5=0,\\ a_6=-621\alpha-\frac{1148}{867}, & a_7=245\alpha+\frac{821}{1734}.\end{cases}$$

$$\begin{cases}b_0=-2352\alpha-\frac{2233}{1156}, & b_1=14168\alpha-\frac{120407}{6936},\\ b_2=-36000\alpha-\frac{13006}{289}, & b_3=47880\alpha+\frac{127575}{2312},\\ b_4=-35640\alpha-\frac{128695}{3468}, & b_5=14952\alpha+\frac{32683}{2312},\\ b_6=-3309\alpha-\frac{2492}{867}, & b_7=301\alpha+\frac{1687}{6936}.\end{cases}$$

$$\begin{cases}c_0=-47280\alpha-\frac{8505}{1156}, & c_1=133336\alpha+\frac{42525}{2312},\\ c_2=-157632\alpha-\frac{5670}{289}, & c_3=101640\alpha+\frac{1575}{136},\\ c_4=-38720\alpha-\frac{4725}{1156}, & c_5=8736\alpha+\frac{1995}{2312},\\ c_6=-1083\alpha-\frac{175}{1734}, & c_7=57\alpha+\frac{35}{6936}.\end{cases}$$

$$\begin{cases} d_0 = -12288\alpha, & d_1 = 22528\alpha, \\ d_2 = -17664\alpha, & d_3 = 7680\alpha, \\ d_4 = -2000\alpha, & d_5 = 312\alpha, \\ d_6 = -27\alpha & d_7 = \alpha. \end{cases}$$

where α is the kernel parameter.

3. EXPERIMENTAL RESULTS AND ANALYSIS

3.1 Experiment

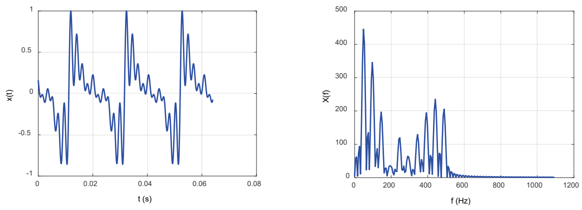
For the purpose of comparative analysis of the interpolation precision, when applying interpolation kernels of the fifth and seventh order, the Experiment was realized. Interpolation is applied over the mathematical Sine signals. The fundamental frequency of the Sine signals corresponds to the tones G1 - G7. The *MSE* was used as a measure of the interpolation precision. The kernels parameter α were optimized by minimizing the *MSE* ($\alpha_{opt} = \arg\min_{\alpha}(MSE)$). By comparing min(*MSE*), the kernel with higher interpolation precision was determined.

3.2 Base

The Base is formed from mathematical Sine test signals. The sine test signal is defined by:

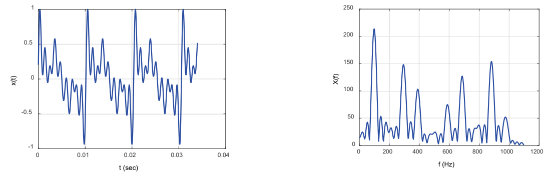
$$s(t) = \sum_{i=1}^{K} a_i \sin(2\pi i f_0 t) \tag{3}$$

where f_0 is the fundamental frequency, a_i is the amplitude of the i-th harmonic, and K is the number of harmonics. The Sine Test signals are created with fundamental frequencies corresponding to music tones G1 (SinG1, f_0 = 49Hz), G2 (SinG2, f_0 = 98Hz), G3 (SinG3, f_0 = 196Hz), G4 (SinG4, f_0 = 392Hz), G5 (SinG5, f_0 = 783.99Hz), G6 (SinG6, f_0 = 1567.98Hz), G7 (SinG7, f_0 = 3135.96Hz) with K = 10. The Sine Test signals are archived on the hard disk in the form of the **wav** files. Sine test signals, corresponding to tones G1 - G7, are shown in: a) time (figs. 1.a - 7.a) and b) spectral (figs. 1.b - 7.b) domain.



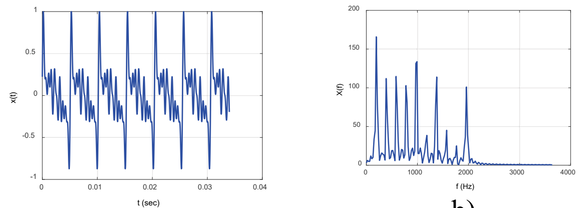
a) b)

Fig. 1. *Sine test signal SinG1, (f_0 = 49 Hz): a) time and b) spectral domain.*



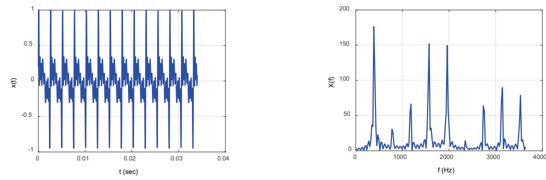
a) b)

Fig. 2. *Sine test signal SinG2, (f_0 = 98 Hz): a) time and b) spectral domain.*



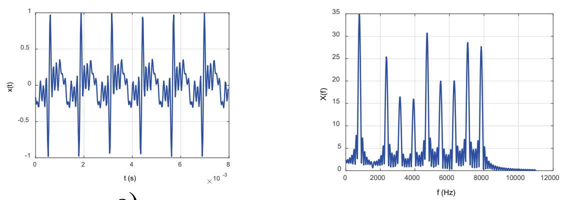
a) b)

Fig. 3. *Sine test signal SinG3, (f_0 = 196 Hz): a) time and b) spectral domain.*



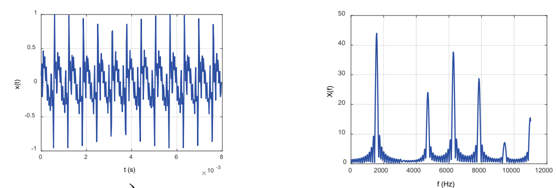
a) b)

Fig. 4. *Sine test signal SinG4, (f_0 = 392 Hz): a) time and b) spectral domain.*



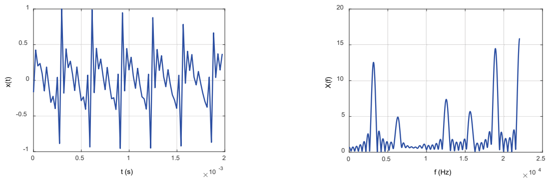
a) b)

Fig. 5. *Sine test signal SinG5, (f_0 = 783.99 Hz): a) time and b) spectral domain.*



a) b)

Fig. 6. *Sine test signal SinG6, (f_0 = 1567.98 Hz): a) time and b) spectral domain.*



a) b)

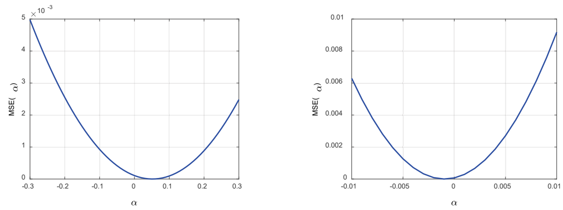
Fig. 7. *Sine test signal SinG7, (f_0 = 3135.96 Hz): a) time and b) spectral domain.*

3.3 Results

The MSE_{min} and the optimal kernel parameters, α_{opt}, for the interpolation kernel of the fifth and seventh order, are shown in Table 1. In addition, Table 1 shows the mean of the optimal parameters α_{opt} and the mean of the MSE_{min}, for both types of interpolation kernels. Graphs of $MSE(\alpha)$ are shown in: a) fig. 8 (SinG1), b) fig. 9 (SinG2), c) fig. 10 (SinG3), d) fig. 11 (SinG4), e) fig. 12 (SinG5), f) fig. 13 (SinG6), and g) fig. 14 (SynG7).

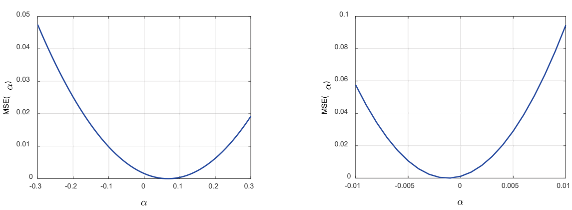
Table 1. *Optimal parameters α_{opt} and MSE_{min} for fifth- and seventh-order interpolation kernels.*

	Fifth-order		*Seventh-order*	
Ton	α_{opt}	MSE_{min}	α_{opt}	MSE_{min}
SinG1	0.050	1.5991*10^-7	-0.001	2.8312*10^-7
SinG2	0.070	1.8420*10^-5	-0.001	4.9448*10^-5
SinG3	0.100	0.0097	-0.002	0.0096
SinG4	-0.080	0.1519	0.002	0.1532
SinG5	-0.230	0.0976	0.005	0.0976
SinG6	0.080	0.1060	-0.002	0.1047
SinG7	-0.470	0.0803	0.010	0.0863
	$\overline{\alpha_{opt\_5}}$	$\overline{MSE_{min\_5}}$	$\overline{\alpha_{opt\_7}}$	$\overline{MSE_{min\_7}}$
	-0.0686	0.0636	0.0016	0.0645



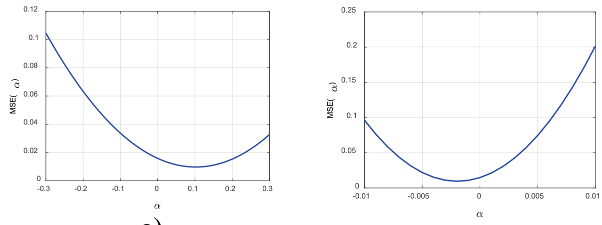
a) b)

Fig. 8. *MSE(α) for interpolation kernels: a) fifth-order and b) seventh-order, for SinG1 Test signal.*



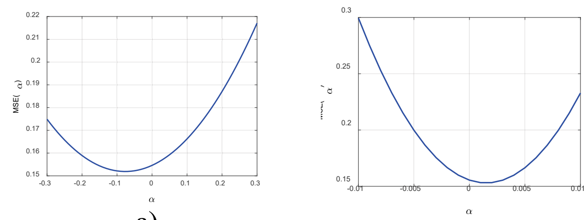
a) b)

Fig. 9. *MSE(α) for interpolation kernels: a) fifth-order and b) seventh-order, for SinG2 Test signal.*



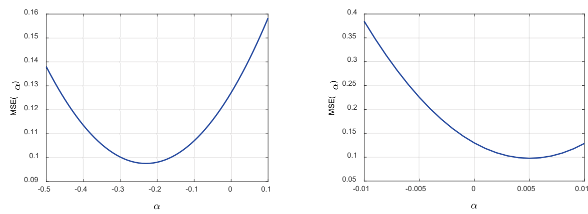
a) b)

Fig. 10. *MSE(α) for interpolation kernels: a) fifth-order and b) seventh-order, for SinG3 Test signal.*



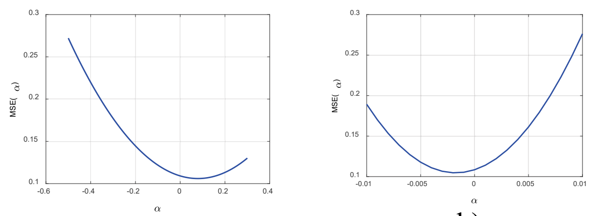
a) b)

Fig. 11. *MSE(α) for interpolation kernels: a) fifth-order and b) seventh-order, for SinG4 Test signal.*



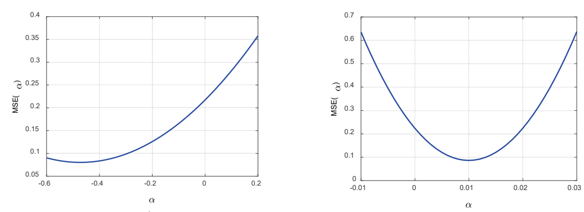
a) b)

Fig. 12. *MSE(α) for interpolation kernels: a) fifth-order and b) seventh-order, for SinG5 Test signal.*



a) b)

Fig. 13. *MSE(α) for interpolation kernels: a) fifth-order and b) seventh-order, for SinG6 Test signal.*



a) b)

Fig. 14. *MSE(α) for interpolation kernels: a) fifth-order and b) seventh-order, for SinG7 Test signal.*

3.4 Analysis of results

Based on the results shown in Table 1 and figs. 8 - 14 it is concluded that:

a) the range of optimal values of the parameter α for the fifth-order kernel is $\alpha_{opt} \in [-0.470, 0.100]$. The mean value of the kernel parameter of the fifth order is $\overline{\alpha_{opt\_5}}$ = -0.0686.

b) the range of optimal values of the parameter α for the seventh-order kernel is $\alpha_{opt} \in [-0.002, 0.001]$. The mean value of the kernel parameter of the fifth order is $\overline{\alpha_{opt\_7}}$ = 0.0016.

c) the mean square error of the 1P interpolation seventh-order kernel compared to the mean square error of the 1P fifth-order kernel is MSE_7 / MSE_5 = 0.0645/0.0636 = 1.014 times higher.

Based on the comparative analysis of the mean square error for: a) the 1P interpolation fifth-order kernel and b) the 1P interpolation seventh-order kernel, it is concluded that the1P interpolation fifth-order kernel is more accurate in relation to the 1P interpolation seventh-order kernel when interpolating Sine Test signals.

4. CONCLUSION

In this paper, a comparative analysis of interpolation precision, in the interpolation of the Sine Test signal, using the 1P interpolation fifth-order and seventh-order kernels, was performed. Higher precision of interpolation was achieved by using the fifth-order interpolation kernel. In contrast to interpolation with Audio signals (Test signals were recorded at a concert piano), where the seventh-order kernel showed higher precision in relation to the fifth-order kernel, in the case of Sine signal interpolation, the seventh-order kernel showed lower precision. The explanation for the greater precision of the fifth-order kernel should be sought in the mathematical definition of the Sine test signal, which has a stronger autocorrelation.

REFERENCE

[1] E. Mejering, K. Zuiderveld, M. Viegever, ’’Image Reconstruction by Convolution with Simetrical Piecewise n-th-Order Polynomial Kernels’’, IEEE Transactions on Image Processing, vol. 8, no. 2, pp. 192-201, Feb. 1999.

[2] H. S. Pang, S.J. Baek, K.M. Sung, ’’Improved Fundamental Frequency Estimation Using Parametric Cubic Convolution’’, IEICE Trans. Fundamentals, vol. E83-A, No. 12, pp. 2747-2750, Dec. 2000

[3] R. G. Keys, ’’Cubic convolution interpolation for digital image processing’’, IEEE Trans. Acout. Speech, & Signal Processing, vol. ASSP-29, pp. 1153-1160, Dec. 1981.

[4] N. Savić, Z. Milivojević, B. Prlinčević, ’’Development of the 2P fifth-degree interpolation convolutional kernel’’, International Journal of Innovative Research in Advanced Engineering (IJIRAE), 8(11), 306-311, 2021.

[5] N. Savić, Z. Milivojević, ’’Optimization of the 3P Keys Kernel Parameters for Interpolacion of Audio Signals’’, International Scientific Conference UNITECH' 20, (pp. 200 – 205), Gabrovo, Bulgaria, Nov. 2020.

[6] N. Savić, Z. Milivojević, B. Prlinčević, & D. Kostić, ’’Septic-convolution Kernel - Comparative Analysis of the Interpolation Error’’, 16th International Conference on DEVELOPMENT AND APPLICATION SYSTEMS, Suceava, Romania, 2022.